Evaluation Of Integrals Involving Orthogonal Polynomials:

Laguerre

Polynomial And Bessel Function Example

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Summary

Using the theory of orthogonal polynomials, their associated recursion relations and differential formulas we develop a new method for evaluating integrals that include orthogonal polynomials. The method is illustrated by obtaining the following integral result that involves the Bessel function and associated Laguerre polynomial:

\[ \int_{0}^{\infty} x^v e^{-x/2} J_v(\mu x)L_n(2v)(x)\,dx = 2^v \Gamma(v + 1/2) \frac{1}{\sqrt{\pi \psi}} \sin \theta(v+1/2) C_n(v)+(1/2) (\cos \theta), \]

where \( \mu \) and \( v \) are real parameters such that \( \mu \geq 0 \) and \( v > -1/2 \), \( \cos \theta \mu = 2-1/4/\mu + 1/4 \), and \( C_n(\lambda)(x) \) is a Gegenbauer (ultraspherical) polynomial. (c) 2006 Elsevier Ltd. All rights reserved.

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